

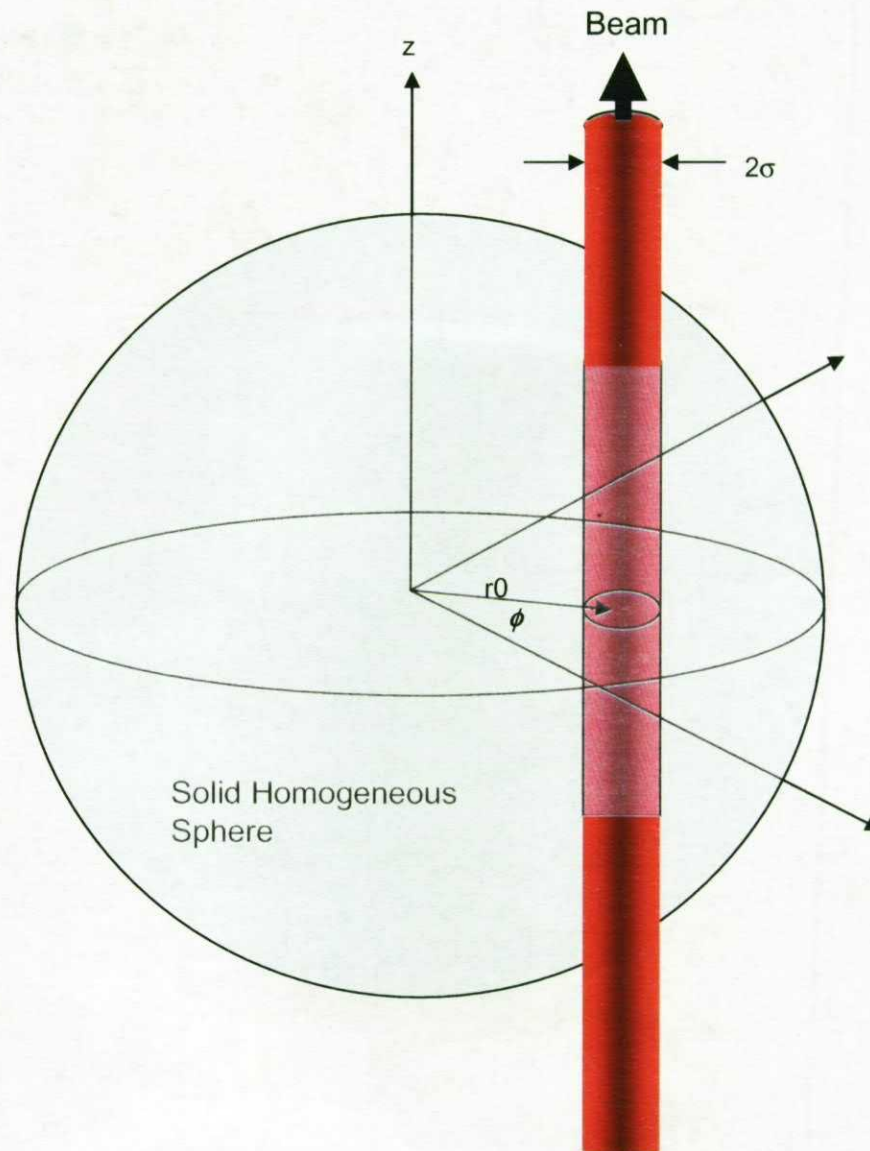
Effects of Beam-Object misalignment on Relative Aerial Density Measurements of Solid Homogeneous Spherical objects using Proton Transmission at PRAD

- Estimate the expected systematic errors in the measurement of relative aerial densities of two static/dynamic objects.
- Chief source of systematic errors is beam-object misalignment for spherical objects.
- For homogeneous spherical objects, the required alignment tolerance scales with object radius, and depends on the dimensions of the beam distribution.

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The Problem: Center of Beam Offset from center of sphere by r_0



Define the Uncertainty

The effective $\rho.dL$ sampled by a beam $B(r,\phi,r_0,\sigma)$ on a sphere of radius R_s is given by:

$$\langle \rho.dL(\sigma, r_0) \rangle = \frac{\iiint \rho(r, \phi, z) B(r, \phi, r_0, \sigma) r dr d\phi dz}{\iint B(r, \phi, r_0, \sigma) r dr d\phi}$$

Where r_0 is the offset of the beam center with respect to the center of the spherical object. The variation in $\rho.dL$ due to the offset is given by

$$\delta \langle \rho.dL(\sigma, r_0) \rangle = \frac{\langle \rho.dL(\sigma, r_0) \rangle - \langle \rho.dL(\sigma, 0) \rangle}{\langle \rho.dL(\sigma, 0) \rangle}$$

Types of Beam Shapes Considered

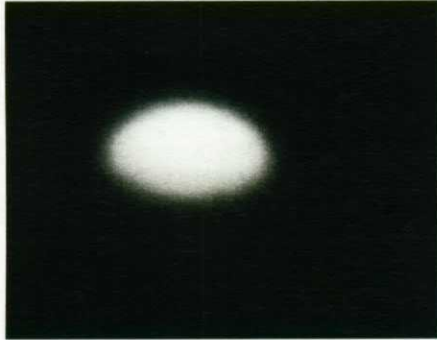
Three types of beams considered on a homogeneous solid sphere.

1. Uniform circular beam of radius σ
2. Gaussian beam of rms width σ
3. An experimentally measured pencil beam developed on April 19, 2004.

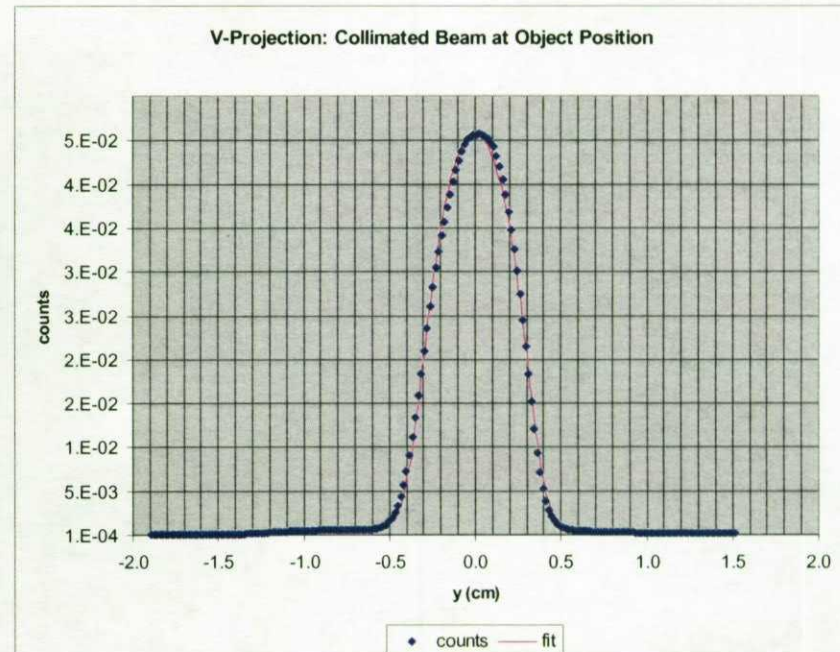
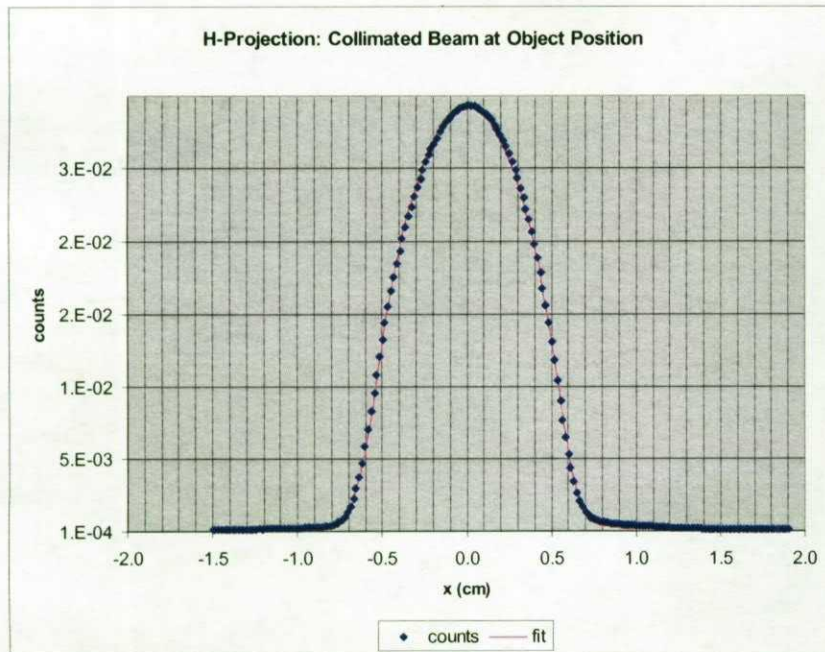
For 1 and 2, we estimate, as function of σ , the value of r_0 required to achieve a given uncertainty in $\rho.dL$.

For 3, we estimate the same quantity at the measured rms width for the X and Y projections (shown in next slide).

Measured Pencil Beam



X and Y projections of the beam are fitted (red solid lines) by a sum of Gaussian and higher order functional forms. Each distribution is assumed to be axially symmetric about its center.



Alignment Tolerance

- We wish to estimate the alignment tolerance r_0 necessary to achieve 0.1%, 0.2% and 0.25% accuracy in measuring relative $\langle \rho.dL \rangle$ for the three beam types.
i.e. determine the value of r_0 for which

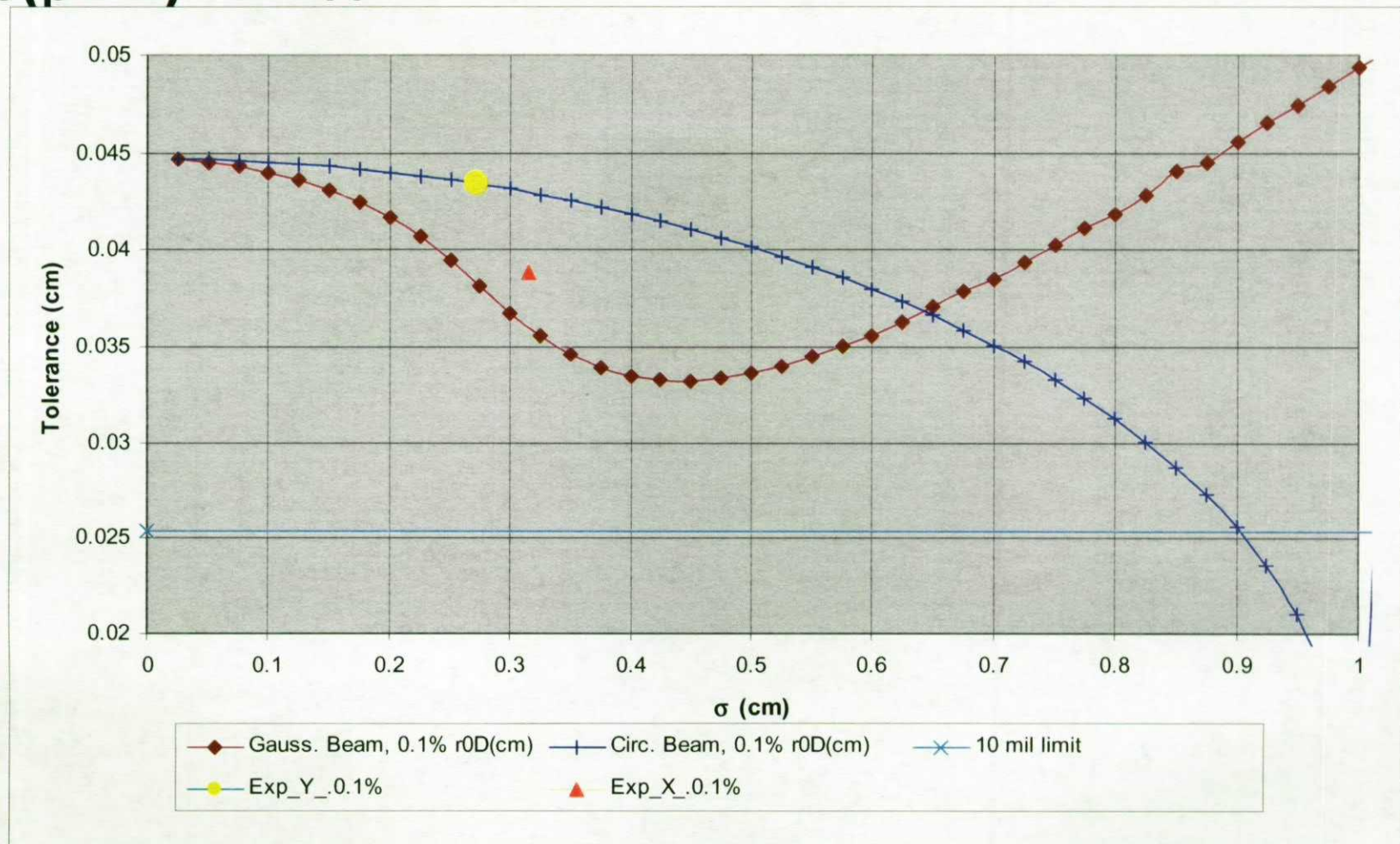
$$\delta \langle \rho.dL(\sigma, r_0) \rangle = 0.1\%, 0.2\% \text{ or } 0.25\%$$

σ is a characteristic dimension of the beam distribution

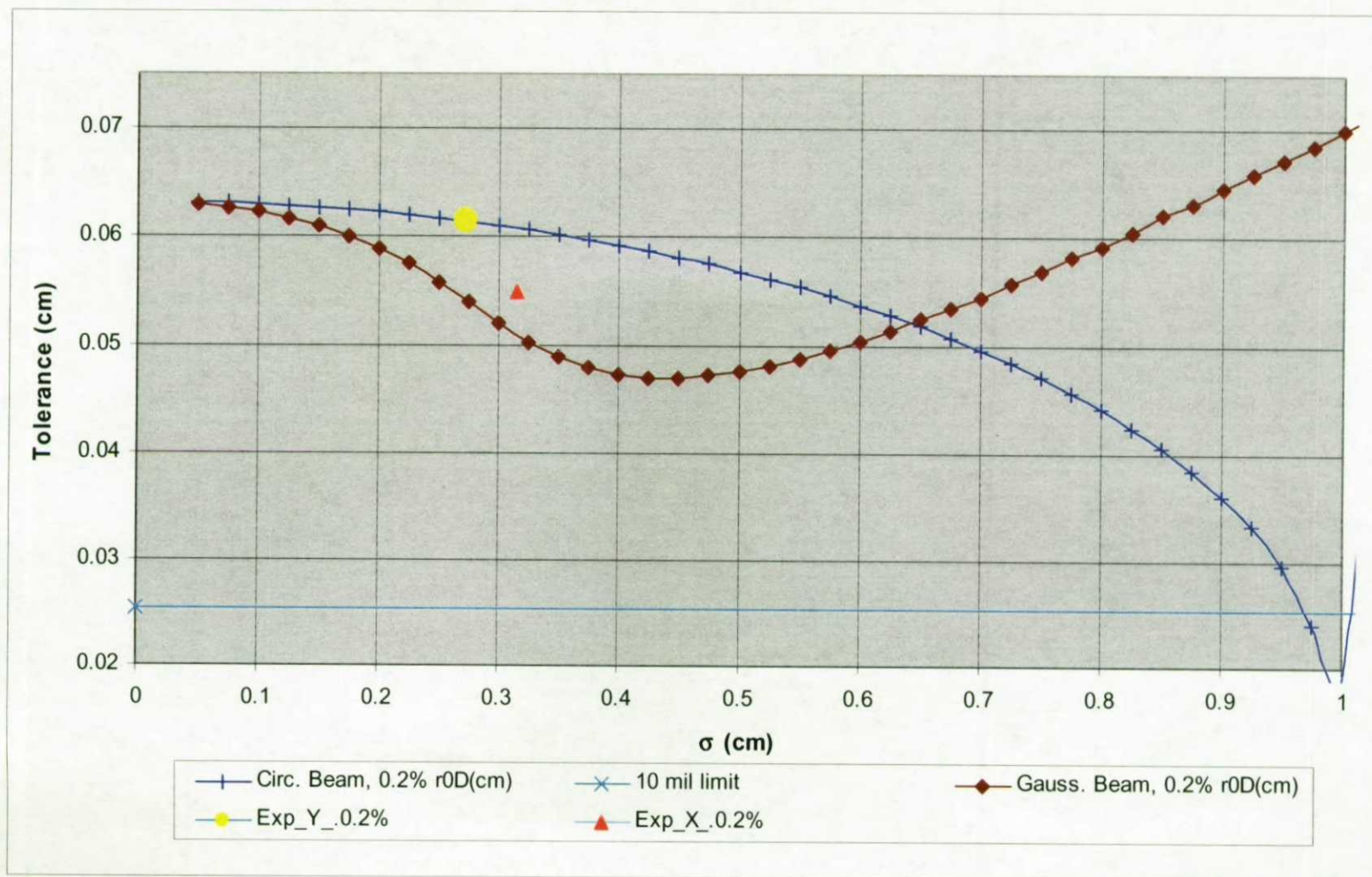
- a) for Gaussian beam σ is the rms width of the distribution
- b) for a uniform circular beam σ is the radius of the distribution
- c) In the case of the measured pencil beam σ is the rms width of the projected distributions.

Estimated Tolerances for uniform and Gaussian beams vs beam size. The yellow and red points are for the vertical and horizontal projections of the experimental pencil beam; $R_{\text{object}} = 1\text{cm}$

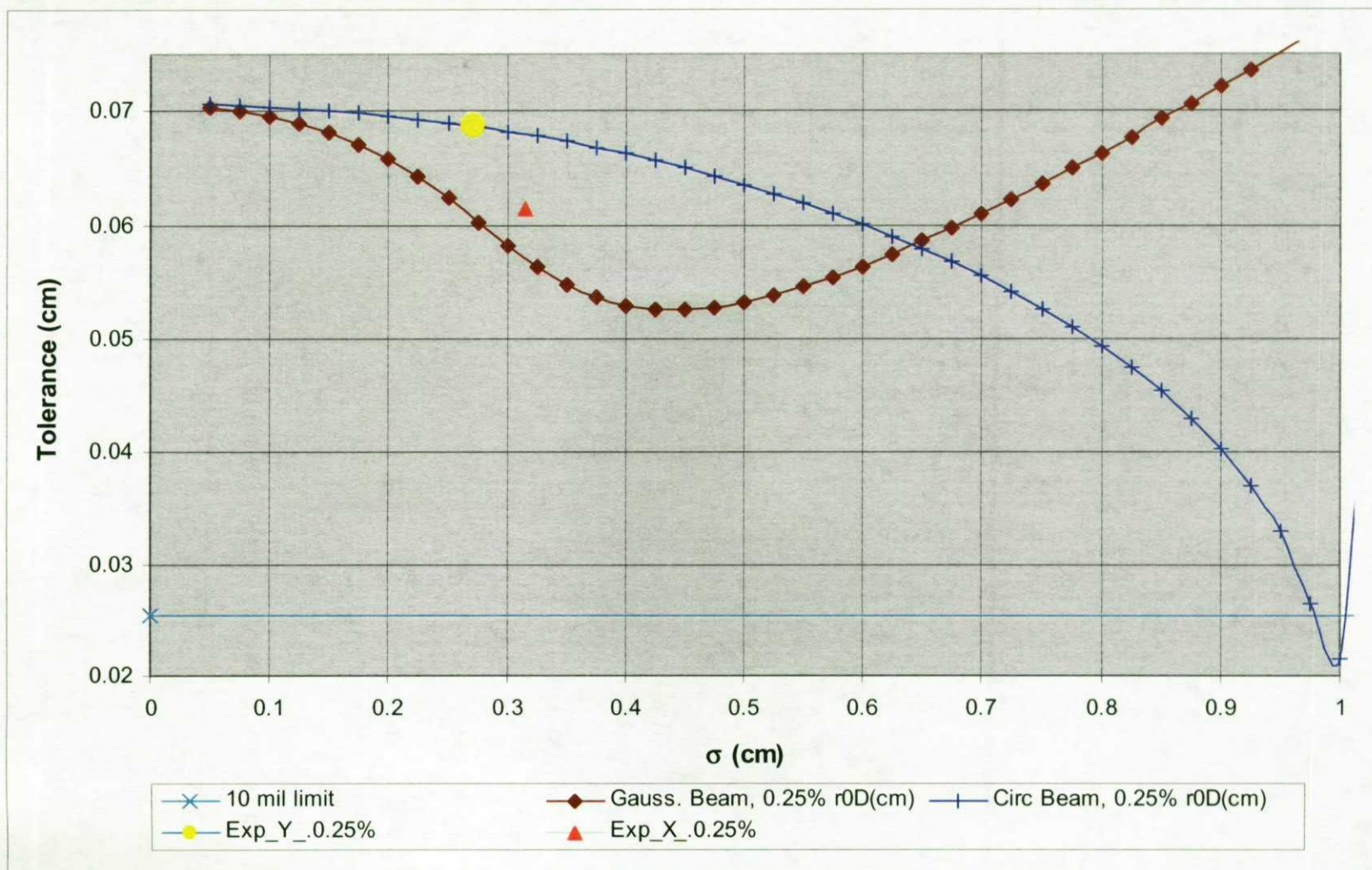
$$\delta(\rho\text{DL}) = 0.1\%$$



$$\delta(\rho DL)=0.2\%$$



$$\delta(\rho DL)=0.25\%$$



Summary

- Avoid using Gaussian beams with beam widths of the order ~ 0.4 to 0.5 the object radius, since such choices lead to more stringent alignment requirements.
- For a 1cm object radius, alignment of 10 mils (which is doable) would contribute less than 0.1% error in aerial density measurements. The required alignment scales with the object radius.
- A round pencil beam of the order of ~ 0.2 to 0.3 of the object radius can be developed perhaps using the present 1 cm diameter collimator. Such beam seems adequate to achieve relative density measurements with uncertainties of less than 0.1%